

State space:

* Representation (Model)

Controllable form

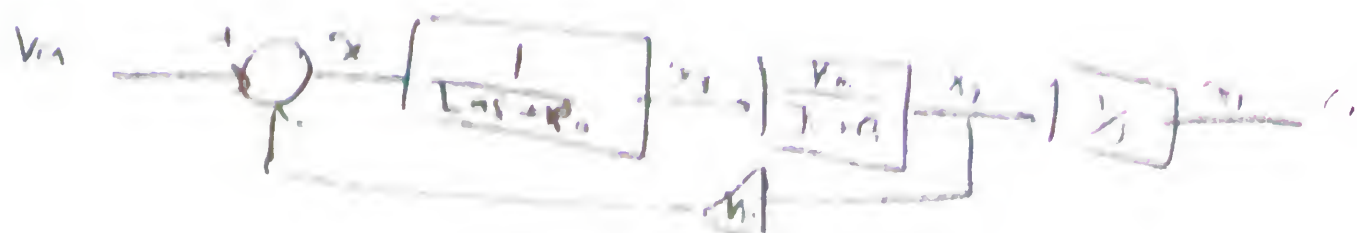
Observable form

Diagonal form

* Analysis

* Design

Ex.



$$\frac{x_1}{x_2} = \frac{1}{s}$$

$$\dot{x}_1 = x_2$$

$$\frac{x_2}{x_3} = \frac{K_m}{J s + B}$$

$$J s x_2 + B x_2 = K_m x_3$$

$$J \dot{x}_2 + B x_2 = K_m x_3$$

$$\dot{x}_2 = \frac{K_m}{J} x_3 - \frac{B}{J} x_2$$

$$\frac{L_a s x_3}{x_2} = \frac{1}{V_a - K_b x_2}$$

$$V_a - K_b x_2 = \frac{1}{L_a s + R_a}$$

$$L_a s x_3 + R_a x_3 = V_a - K_b x_2$$

$$L_a \dot{x}_3 + R_a x_3 = V_a - K_b x_2$$

$$\dot{x}_3 = \frac{V_a}{L_a} - \frac{K_b}{L_a} x_2 - \frac{R_a}{L_a} x_3$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{B}{J} & \frac{K_m}{J} \\ 0 & -\frac{K_b}{L_a} & -\frac{R_a}{L_a} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L_a} \end{bmatrix} V_a$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

form

$$\dot{x} = A x + B u$$

$$y = C x$$

to make the system state space to be in the controllable form, then if any change in the i/p then the states should change.

So the B matrix should be a non-zero element matrix

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Special case from controllable form is the canonical controllable form

Observable

if the C matrix of the system is non zero element matrix then the form is said to be Observable form.

Diagonal form

$$A = \begin{bmatrix} \diagup & & \\ & \circ & \\ & & \circ \end{bmatrix}$$

↑ poles of the system

Diagonal form make the system is easy to analyse the stability

Analysis of state space ..

$$\dot{X} = AX + Bu \xrightarrow{\text{and}} y = CX$$

$$\Downarrow \mathcal{L}$$
$$sX = AX + Bu$$

$$(sI - A)X = Bu$$

$$X = (sI - A)^{-1} Bu$$

$$y = CX$$

$$= C(sI - A)^{-1} Bu$$

$$\therefore \frac{y}{u} = C(sI - A)^{-1} B$$

$$C/Cq \therefore |sI - A| = 0$$

the system have ^{Actual} C.L poles locations

* Design problem

$$\dot{X} = Ax + Bu \quad \text{if } u = -Kx$$

$$\dot{X} = Ax - BKx$$

$$Sx = (A - BK)x$$

$$(S - A + BK)x = 0$$

$$[SI - A + BK] = 0$$

Actual ~~o~~ poles

$$cfeq \quad |SI - A| = 0$$

if the desired c/c eq. $\alpha(s)$.

(this topic ~~is~~ will be detailed next section)